Plane of Symmetry

Plane of Symmetry is defined as an imaginary Plane that bisects the molecule in such a way that the two Parts are mirror images of each other.

It should be noted that the operation of reflection gives a configuration equivalent to the original one. If the operation is carried out twice on the molecule, we get the original configuration.

i.e. $\sigma . \sigma . = \sigma^2 = E$

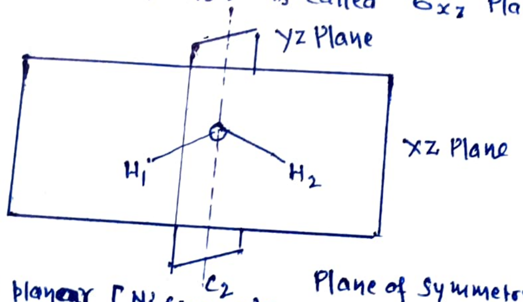
The plane of symmetry can be classified into three types:-

(a) Vertical Plane (σ_v) :- The plane passing through the principal axis and one of the subsidiary axis (if present) is called Vertical Plane.

(b) Horizontal Plane (σ_h) :- The plane which is perpendicular to the principal axis is called Horizontal Plane.

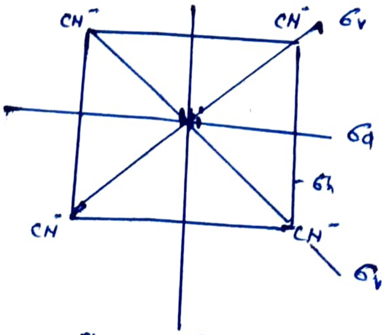
(c) Dihedral Plane (σ_d) :- The plane passing through the principal axis but bisecting an angle between two subsidiary axis (C_2) is called dihedral plane.

Now consider water (H_2O) molecule. It has two symmetry planes i.e. σ_{xz} & σ_{yz}. One is passing through oxygen atom and bisecting the angle $\angle HOH$ called as σ_{yz} Plane. The other plane of symmetry is passing through oxygen atom and two H-atom is called σ_{xz} Plane.



Plane of Symmetry in H_2O molecule.

In case of square planar $[Ni(CN)_4]^{2-}$ there are four σ_v Planes and one σ_h Plane. Two σ_v Planes pass through C_4 axis, Ni(II) ion and two CN^- ions at opposite corners. Two σ Plane pass through C_4 axis, Ni(II) ion and between two CN^- and is called σ_d. The molecular plane passing through Ni(II) ion and four CN^- ion is called σ_h



Similarly in hexagonal Planar Benzene molecule, Six σ_v and one σ_h are Present.

Improper axis of Symmetry or Rotational - reflection axis of Symmetry (S_n) :-

This operation is combination of a rotation (C_n) with reflection (σ) in a plane perpendicular to the rotational axes. After this composite operation, it leaves the molecule in an indistinguishable configuration.

$$S_n = C_n \cdot \sigma_h$$

If any molecule contains C_n and σ_h operations, then it is generally contains S_n.

$$S_2 = C_2 \cdot \sigma_h = i$$

S_2 is i because after the rotation by 180° and then reflection in a plane perpendicular to C_2 produce i.

$$S_3 = C_3 \cdot \sigma_h$$

BCl_3 contains S_3. BCl_3 molecule after C_3 and then $\sigma_h \perp C_3$ produce indistinguishable configuration.

As C_n generates n operations i.e $C_n^1, C_n^2, C_n^3 \ldots C_n^n = E$ and S_n also generates n such operations when n is even but generates $2n$ operations when n is odd.

If n = odd

i.e. $n = 3$

$S_3^1 = C_3^1 \cdot \sigma_h^1 = C_3 \cdot \sigma_h$

$S_3^2 = C_3^2 \, \sigma_h^2 = C_3^2 \cdot E = C_3^2$

$S_3^3 = C_3^3 \, \sigma_h^3 = E \cdot \sigma_h^2 \cdot \sigma_h = E \cdot E \cdot \sigma_h = \sigma_h$

We know
$\left[\begin{array}{l} C_3^3 = E \\ \sigma \cdot \sigma = \sigma^2 = E \end{array} \right]$

$S_3^4 = C_3^4 \cdot \sigma_h^4 = C_3^3 \cdot C_3^1 \cdot \sigma_h^2 \cdot \sigma_h^2 = E \cdot C_3^1 \cdot E \cdot E = C_3^1$

$S_3^5 = C_3^5 \cdot \sigma_h^5 = C_3^3 \cdot C_3^2 \cdot \sigma_h^2 \cdot \sigma_h^2 \cdot \sigma_h = E \cdot C_3^2 \cdot E \cdot E \cdot \sigma_h = C_3^2 \sigma_h$

$S_3^6 = C_3^6 \, \sigma_h^6 = C_3^3 \cdot C_3^3 \cdot \sigma_h^2 \cdot \sigma_h^2 \cdot \sigma_h^2 = E \cdot E \cdot E \cdot E \cdot E = E$

Group

A group is a Collection of elements that are interrelated according to Certain rule.
We are Concerned here with the groups formed by the sets of symmetry operations that may be Carried out on the molecule or crystals.

The characteristics of a mathematical group are :-

(a) Closure
(b) Identity
(c) Inverse
(d) Association

(a) Closure :-
The product of any two elements in the group and the square of each element must be an element in the group.

The Product of any two elements A and B Produce C. So C must be element of the group.

$$A \cdot B = C$$

$$A^2 = D$$

$$B^2 = E$$

then C, D and E must be elements of the group.

The order of Combination is very important as AB is not necessarily equal to BA.

If $AB = BA$, the members A and B are said to 'Commute'.

and if $AB \neq BA$ the members A and B are not Commutative.

The members of the group which are Commutative from Abelian group.

(b) Identity:- One element of the group must Commute with all other elements and leave them Unchanged.
This element is Called Identity and represented as E.

Identity must be present in a group

$$E \cdot A = A \cdot E = A$$

$$E \cdot B = B \cdot E = B$$

A and B are elements of the group.

(c) Inverse: –

Every member of the group must have its inverse as an member of the group.

$$A \cdot A^{-1} = A^{-1} \cdot A = E$$

(d) Association: –

Multiplication of elements of a group must be associative.

$$A(B \cdot C) = (A \cdot B) \cdot C$$

Symmetry elements of a molecule constitute a group.

Point Group: –

A Point groups is defined as a set of Symmetry operations (rotation, reflection etc.) that leave an object or molecule unchanged with all operations passing through a fixed point.

Multiplication table for C_{2v} Point group – Water (H_2O) belongs to C_{2v} Point group.